An Efficient Distributed Online Algorithm to Detect Strong Conjunctive Predicates

Loon-Been Chen
Dept. Information Management
Chin-Min College
lbchen@mis.chinmin.edu.tw

I-Chen Wu
National Chiao-Tung University
icwu@csie.nctu.edu.tw

Abstract
Detecting strong conjunctive predicates is a fundamental problem in debugging and testing distributed programs. A strong conjunctive predicate is a logical statement to represent the desirable event of the system. Therefore, if the predicate is not true, an error may occur because the desirable event do not happen. Recently proposed detection algorithms have the problem of unbounded state queue growth since the system may generate a huge amount of execution states in a very short time. In order to solve this problem, this paper introduces the notion of removable states that can be disregarded in the sense that detection results still remain correct. A fully distributed algorithm is developed in this paper to perform the detection in an online manner. Based on the notion of removable states, the time complexity of the detection algorithm is improved as the number of states to be evaluated is reduced.

Keywords: Conjunctive predicate, distributed debugging, distributed system, global predicate detection.

1 Introduction
With the rapid development of networks and distributed systems, programming in distributed environments is becoming more common. However, the difficulty of distributed programming is much higher than that of sequential programming. This is because distributed debugging requires the ability to analyze and control the execution of processors that run asynchronously. Also, within a distributed environment, stopping a program at a specific breakpoint is non-trivial.

It is well understood that distributed programs are usually designed to obey certain conditions [6]. For example, a distributed mutual exclusion program obeys the condition "at any time, the number of processes in the critical section is no more than 1". If it violates this condition, an error (two or more processes are in the critical section simultaneously) may occur. Typically, the conditions are formulated as boolean expressions, called global predicates [1, 5]. Detecting whether a given global predicate is satisfied is essential for debugging and testing distributed computations.

As the detection of general global predicate was proved to be NP-complete [2], most researchers restricted their research regarding a specific class of global predicates to detect in polynomial time. In this paper, the focus is on an important class of global predicates, known as conjunctive predicate [7, 8, 11, 12], which can be expressed as a conjunctive form of local predicates. The local predicate is a boolean expression defined by the local variables of the process. At any time, a process can evaluate its local predicate without communication.

In this paper, the problem of detecting whether a given conjunctive predicate $\Phi$ is definitely true [5, 12] is considered. $\Phi$ is definitely true if for all runs of the distributed program, $\Phi$ is true at some time. Intuitively, detecting this sort of global predicates is used to ensure that a certain desirable event occurs. For simplicity, we define predicate DEFINITELY($\Phi$) is true if and only if $\Phi$ is definitely-true. DEFINITELY($\Phi$) is called strong conjunctive predicate in [8].

In [12], Venkatesan and Dathan proposed a distributed algorithm to detect DEFINITELY($\Phi$). This algorithm performs an offline evaluation of the predicates, i.e. predicates are evaluated after the program execution is terminated. The analysis shows that their algorithm uses $O(p^3M_t)$ ad-
ditional control messages with the size of each being only $O(1)$, where $p$ is the number of processes and $M_t$ is the total number of truth value changes of the local predicates. In [8], Garg and Waldecker proposed an algorithm that evaluates the predicate in an online manner, i.e. predicates are evaluated immediately following each instruction execution. This algorithm employs a central debugger that collects debug information from application processes and then performs the detection. The time complexity of the detection algorithm is $O(n)$, where $n$ is the maximum number of states in one application process. Compared with Venkatesan and Dathan’s algorithm, this algorithm uses only additional control messages with the size of each being $O(1)$, where $n$ is the total number of messages that all application processes receive.

One disadvantage of the above algorithms is that the debugger evaluates execution states, which are collected from application processes, in a certain order. Restated, before evaluating certain states, all other states are queued. Since real systems can generate hundreds of states in a very short time, the queues may grow unbounded. To solve this problem, in this paper, the notion of removable states is introduced. By discarding the removable states, the space requirement for each process can be minimized to $O(p)$, where $p$ is the number of processes. While minimizing the memory space, time complexity is also improved to $O(n)$, because the number of states to be evaluated is reduced. Our algorithm does not require exchange of control messages during program execution, because all the debug information is piggybacked in normal application messages.

The remainder of this paper is organized as follows. In Section 2, we define model and notations. In Section 3, we introduce the notion of removable states and derive the condition of identifying removable states. Based on this result, Section 4 presents an efficient way to maintain non-removable states, and then discusses a new detection algorithm. Finally, a concluding remark is made in Section 5.

2 Model and Notations

A distributed system consists of $p$ processes denoted by $P_1, P_2, \ldots, P_p$. These processes share no global memory and no global clock. Message passing is the only way for processes to communicate. The transmission delay of the communication channel between each pair of processes is random. However, we assume that no message in any channel is lost, altered, or spuriously introduced.

States and Events At a given time, the state of a process is defined by its variable’s values. The states of processes can change only when events are executed. There are three kinds of events: an internal event which does a local computation, a send event which sends a message to another process, and a receive event which receives a message from another process via the channel.

The $x^{th}$ event occurring in process $P_i$ is referred to as $e_{i,x}$. The number $x$ is called the sequence number of $e_{i,x}$. Figure 1 illustrates the events of the execution of a distributed program. Event $e_{i,x}$ happens before event $e_{j,y}$, denoted by $e_{i,x} \rightarrow e_{j,y}$, if and only if one of the following conditions holds [9]:

1. $i = j$ and $x < y$.
2. A message is sent from $e_{i,x}$ to $e_{j,y}$.
3. Another event $e_{k,z}$ exists such that $e_{i,x} \rightarrow e_{k,z}$ and $e_{k,z} \rightarrow e_{j,y}$.

In this paper, the system is assumed to recognize the happen-before relationships by using vector clocks [10] (Figure 1). With this approach, each process $P_i$ maintains an integer vector $vector_i[1..p]$. Initially, each process $P_i$ sets $vector_i$ to $[0,0,\ldots,0]$ but $vector_i[1] = 1$. When a process $P_i$ executes an internal event, it increases $vector_i[i]$ by 1. When process $P_i$ sends out a message, it increases $vector_i[i]$ by
A local predicate is a boolean expression of the process states. At any time, the process can evaluate its local predicate without communication. A global predicate is a boolean expression, which involves the states of several processes. In this paper, we consider an important class of global predicates, known as conjunctive predicate, which can be expressed in a conjunctive form $LP_1 \land LP_2 \land \ldots \land LP_p$, where $LP_i$ is the local predicate of process $P_i$, $i = 1, 2, \ldots, p$. For simplicity, we use either $\Phi$ or $LP_1 \land LP_2 \land \ldots \land LP_p$ to denote the conjunctive predicate.

In [12], Venkatesan and Dathan indicated that in a typical software development environment, developers may have occasion to use the conjunctive predicate $\Phi$ in one or more of the following ways:

- **DEFINITELY($\Phi$) is true iff $\Phi$ is definitely true.** $\Phi$ is definitely true if in every path from the initial node to the final node in the lattice, $\Phi$ holds in some node. Detecting this kind of global predicates is usually used to ensure a certain desirable event occurs. For example, we can consider a distributed two-phase commit protocol (see Figure 3). When the master decides to commit the transaction, it must assure that all the slaves are prepared to commit. Assume that there are two slaves, $P_1$ and $P_2$. Let $\Phi = LP_1 \land LP_2$, where $LP_1 = \{P_1$ is committable$\}$ and $LP_2 = \{P_2$ is committable$\}$. In Figure 3(b), a path (depicted by bold lines) exists in which $\Phi$ is not true in all nodes. Therefore, $\Phi$ is not definitely true. This implies that an error may occur because at least one slave process is not ready to commit during the program execution which corresponds to this bold path.
Figure 3: An example of two-phase commit protocol. Let $\Phi = LP_1 \land LP_2$, where $LP_1 = \{P_1 \text{ is committable}\}$ and $LP_2 = \{P_2 \text{ is committable}\}$. (a) In process $P_1$ ($P_2$), a state is shaded if the local predicate $LP_1$ ($LP_2$) holds within this state. (b) A global state is shaded if $\Phi$ holds (i.e. both $P_1$ and $P_2$ are prepared to commit) within this global state.

Figure 4: (a) Example of intervals. (b) A scenario that $\Phi$ is not definitely true in intervals.

- $POSSIBLY(\Phi)$ is true iff $\Phi$ is possibly true. $\Phi$ is possibly true if a path exists in the lattice such that $\Phi$ holds in some node. Detecting this kind of predicates is usually used to ensure that certain undesirable events do not occur. For example, consider a mutual exclusive program, which runs on a system with two processes $P_1$ and $P_2$. Let $\Phi = \{(P_1 \text{ is in the critical section}) \land (P_2 \text{ is in the critical section})\}$. If $\Phi$ is possible true, the undesirable event (both processes are in the critical section) may occur in some run of the program.

**Intervals** Researchers in [8, 12] proposed a necessary and sufficient condition of whether $DEFINITELY(\Phi)$ holds. This condition uses the notion of intervals. An interval $t$ is a pair of events in the same process, in which $t.lo$ and $t.hi$ are referred to as its beginning event and ending event respectively. Furthermore, event $t.lo$ turns the truth value of the local predicate from false to true, events between $t.lo$ and $t.hi$ do not change the truth value, and event $t.hi$ turns the truth value...
from true to false. Two intervals $t$ and $t'$ are overlapped if $t.lo \rightarrow t'.hi$ and $t'.lo \rightarrow t.hi$. For example, in Figure 4(a), interval $t_2$ and $t_4$ are overlapped but $t_2$ and $t_5$ are not. A set of overlapping global interval (OGI-set) is a set of intervals, one from each process, in which each pair of intervals is overlapped. For example, interval set \{t_2, t_4, t_5\} in Figure 4(a) is an OGI-set.

To simplify, the following notations for two interval sets $\mathcal{I}$ and $\mathcal{J}$ are defined:

- $\mathcal{I} \subseteq \mathcal{J}$: For all $t \in \mathcal{I}$ and $t' \in \mathcal{J}$ in the same process, $t.lo \rightarrow t'.lo$ or $t.lo = t'.lo$. For example, in Figure 4(a), $\{t_2, t_3, t_5\} \subseteq \{t_2, t_4, t_5\}$.
- $\mathcal{I} \rightarrow \mathcal{J}$: For all $t \in \mathcal{I}$ and $t' \in \mathcal{J}$, $t.lo \rightarrow t'.hi$.

Figure 4(b) illustrates an interesting scenario that DEFINITELY($\Phi$) does not hold. In this figure, $t_1$ and $t_2$ are not overlapped since no message exists from $t_1.lo$ to $t_2.hi$. Thus, a program execution can be constructed such that $\Phi$ is true from global states $S_1$ to $S_2$, but is false in $S_4$. Theorem 2.2 generalizes this scenario.

**Theorem 2.2 ([8, 12])** For a distributed program, DEFINITELY($\Phi$) holds if and only if there exists an OGI-set.

**Distributed Online DEFINITELY($\Phi$) Detecting Problem** In a distributed environment, processes collect the execution states of other processes by exchanging messages. In other words, when a process $P_i$ executes an event $e_{i,z}$, all the events (and the associated states) that it can observe are those that happen before $e_{i,z}$. These events are denoted by $E_{i,x}$, i.e., $E_{i,x} = \{e_{j,y} | e_{j,y} \rightarrow e_{i,z} \text{ or } e_{j,y} = e_{i,z}\}$. $E_{i,x}$ is called the E-set of $e_{i,z}$. If $E_{i,x} \subseteq E_{j,y}$ then $E_{j,y}$ is called a future E-set of $E_{i,x}$. The following property can be verified easily:

**P1** Event $e_{i,z} \rightarrow e_{j,y}$ if and only if $E_{i,x} \subseteq E_{j,y}$.

In this paper, the distributed online DEFINITELY($\Phi$) detecting problem is that whenever process $P_j$ executes an event, say $e_{i,z}$, it tests whether DEFINITELY($\Phi$) holds for the debug information associated with E-set $E_{i,z}$.

### 3 Identifying Removable Intervals

According to Theorem 2.2, DEFINITELY($\Phi$) holds if and only if at least one OGI-set exists. To detect DEFINITELY($\Phi$) efficiently, the main idea of this paper is to derive only the minimum OGI-set and treat the others as removable. An OGI-set $I$ in E-set $E_{i,z}$ is minimum if $I \leq I'$ for all OGI-sets $I'$ in $E_{i,z}$. The minimum OGI-set in $E_{i,z}$ is given by $F(E_{i,z})$.

To simplify our presentation, pseudo event and volatile interval are defined in Definition 3.1.

**Definition 3.1** For an E-set $E_{i,z}$, define the volatile interval $\hat{t}$ for each process $P_j$ as follows:

1. If an interval $t$ exists in $P_j$, it satisfies $t.lo \in E_{i,z}$ but $t.hi \notin E_{i,z}$ (e.g. interval $t$ in Figure 5), then $\hat{t}.lo = t.lo$, and $\hat{t}.hi$ is a pseudo event with vector $(\hat{t}.hi) = [\infty, \infty, \ldots, \infty]$.
2. Otherwise, let $e_{j,y} \in E_{i,z}$ be the last event from $P_j$ (e.g. event with vector clock [3, 0, 0] in Figure 5), both $\hat{t}.lo$ and
Theorem 3.1
rem 3.1.

volatile. By this definition, the following property is useful in the remainder of this paper:

An E-set $E_{i,x}$ may not contain an OGI-set. However, $\widehat{E}_{i,x}$ always contains an OGI-set because intervals in $f(\widehat{E}_{i,x})$ are pairwisely overlapped. This is due to $\text{vector}(t,hi) = [\infty, \infty, \ldots, \infty]$ for any interval $t \in f(\widehat{E}_{i,x})$. Intuitively, $f(\widehat{E}_{i,x})$ is a candidate of OGI-sets within future E-sets. The following property is useful in the remainder of this paper:

P2 Let E-set $E_{i,x} = E_{j,y} \cup E_{k,z}$. The events in $E_{i,x}$ is from either $E_{j,y}$ or $E_{k,z}$. Hence, the interval $t$ is nonvolatile in $E_{i,x}$ if and only if $t$ is nonvolatile in either $E_{j,y}$ or $E_{k,z}$.

Given an E-set $E_{i,x}$, intervals are said to be removable if they do not belong to the minimum OGI-sets of all the future E-sets of $E_{i,x}$, because deriving the minimum is our only concern. Specifically, an interval $t \in E_{i,x}$ is $E_{i,x}$-removable if $t \notin F(\widehat{E}_{i,x})$ for all $E_{j,y}$, where $E_{i,x} \subseteq E_{j,y}$ (because the intervals in $E_{j,y}$ are nonvolatile). By this definition, the following property holds:

P3 If interval $t$ is $E_{i,x}$-removable then $t$ is $E_{j,y}$-removable for all $E_{j,y}$ where $E_{i,x} \subseteq E_{j,y}$.

Next, the necessary and sufficient condition to identify removable intervals is derived in Theorem 3.1.

Theorem 3.1

In an E-set $E_{i,x}$, a nonvolatile interval $t$ is $E_{i,x}$-removable if and only if $t \notin F(\widehat{E}_{i,x})$.

Proof. $(\Rightarrow)$ If $t$ is $E_{i,x}$-removable, since $\widehat{E}_{i,x}$ is a future E-set of $E_{i,x}$, $t \notin F(\widehat{E}_{i,x})$ can be derived (Property P3).

$(\Leftarrow)$ Let $E_{j,y}$ be a future E-set of $E_{i,x}$. We will prove this direction by showing that if $t \in F(\widehat{E}_{j,y})$ then $t \in F(\widehat{E}_{i,x})$. As illustrated in Figure 6, partition $F(\widehat{E}_{j,y})$ into $S_1 \cup S_2$, where $S_2 \subseteq \widehat{E}_{i,x}$ and $S_1 = F(\widehat{E}_{j,y}) \setminus S_2$ (note that $S_2 \neq \emptyset$ since $x \in S_2$). Since $S_1 \rightarrow t$, we can derive that there exists a set of volatile intervals in $\widehat{E}_{i,x}$, say $S_1'$, such that the set of beginning events of $S_1$ and $S_1'$ are identical.

The set $S_1' \cup S_2$ is an OGI-set in $\widehat{E}_{i,x}$ since $S_1' \rightarrow S_2$ and $S_2 \rightarrow S_1'$ (because the intervals in $S_1'$ are volatile). Next, we show that $S_1' \cup S_2$ is the minimum OGI-set of $\widehat{E}_{i,x}$ i.e. $S_1' \cup S_2 = F(\widehat{E}_{i,x})$. By contradiction, assume that $S_1' \cup S_2 \nsubseteq F(\widehat{E}_{i,x})$ and $S_1' \cup S_2 \neq F(\widehat{E}_{i,x})$. As shown in Figure 6, partition $F(\widehat{E}_{i,x})$ into $T_1' \cup T_2$, where $T_1'$ and $T_2$ are sets with volatile and nonvolatile intervals, respectively. Through Figure 6, we can derive that $T_1' \subseteq S_1'$ because $T_1' \cup T_2 \subseteq S_1' \cup S_2$ and a volatile interval must be the last interval in the process. Let $T_1$ represent the set containing nonvolatile intervals in $S_1$, as shown in Figure 6. (Note that the set of beginning events of $T_1$ and $T_1'$ are identical.) Then, $T_1 \cup T_2$ is an OGI-set in $E_{j,y}$ because

1. $T_1 \rightarrow T_2$ in $E_{j,y}$ since $T_1' \rightarrow T_2$ in $F(\widehat{E}_{i,x})$.
2. $T_2 \rightarrow T_1$ in $E_{j,y}$ since $T_2 \rightarrow S_2$, $S_2 \rightarrow S_1$, and $T_1 \subseteq S_1$.

Therefore, $T_1 \cup T_2$ is an OGI-set in $E_{j,y}$ and $\{T_1 \cup T_2\} \nsubseteq \{S_1 \cup S_2\}$. This derivation con-
the detection scenarios. In part (a) of this figure, Corollary 3.1

detection algorithms with and without using the notion of removable states. Figure 7 illustrates accurate because removable intervals in the E-set is said to be removable if there exists a future E-set of the process containing the interval and is not contained in any of its previous E-sets. Therefore, the theorem holds.

The following corollary extended from this theorem is useful for the later parts of this paper.

**Corollary 3.1** If E-set \( E_{j,y} \subseteq E_{i,x} \) then \( F(\hat{E}_{j,y}) \leq F(\hat{E}_{i,x}) \).

**Proof.** Based on Theorem 3.1, this corollary is accurate because removable intervals in the E-set \( E_{j,y} \) must be also removable in its future E-sets.

\[
\begin{align*}
E_{j,y} \ (\text{all events happen before } e_{j,y}) \\
p_{j} & \text{piggyback whole } E_{j,y} \text{ in this message} \\
p_{i} & \text{in this message} \\
E_{i,x-1} & \text{piggyback only } F(\hat{E}_{j,y}) \text{ in this message} \\
e_{i,x} & \text{from set } F(\hat{E}_{i,x-1}) \text{ and } F(\hat{E}_{j,y}) \\
\text{Construct } F(\hat{E}_{i,x-1}) & \text{from set } F(\hat{E}_{i,x-1}) \text{ and } F(\hat{E}_{j,y}) \\
\end{align*}
\]

Figure 7: (a) Illustration of the detection without the notion of removable states. (b) Illustration of the detection that discards the removable states.

Theorem 4.1. Assume that process \( P_j \) sends a message, say \( m \), to process \( P_j \), where the send and receive event of \( m \) is \( e_{j,y} \) and \( e_{i,z} \), respectively. The intervals in \( F(\hat{E}_{i,z}) \) must be from either \( F(\hat{E}_{i,z-1}) \) or \( F(\hat{E}_{j,y}) \).

**Proof.** E-set \( E_{i,z} \) is a future E-set of \( E_{i,z-1} \) and \( E_{j,y} \) since \( E_{i,z} = E_{i,z-1} \cup E_{j,y} \). According to Theorem 3.1, the intervals in \( F(\hat{E}_{i,z}) \) must be from either \( F(\hat{E}_{i,z-1}) \) or \( F(\hat{E}_{j,y}) \).

A naive approach to compute \( F(\hat{E}_{i,z}) \) in Figure 7(b) is to apply the algorithms that were proposed in [8, 12] and let \( F(\hat{E}_{i,z-1}) \) be the inputs of the algorithms. Their algorithms work by testing overlap between intervals and remove useless intervals systematically. However, in worst case, in each run it performs \( O(p^2m) \) testing to ensure that all the \( p \) intervals (one from each process) are pairwise overlapped. The total time is \( O(p^2m) \) if the maximum number of events in one process is \( m \). In this section, a more efficient detection algorithm that runs in time \( O(pm) \) is proposed. In Subsection 4.2 we present an efficient approach to maintain the minimum OGI-sets. Based on this result, Subsection 4.3 presents our new detection algorithm and analysis its complexity and correctness.

### 4.2 Maintain Minimum OGI-sets Efficiently

Let \( X, Y, \) and \( Z \) be the E-sets satisfy the condition \( Z = X \cup Y \). This section describes how to derive the minimum OGI-set of \( Z \) by given the minimum OGI-sets of subsets \( X \) and \( Y \). Before describing our approach, some notations used in this section are defined as follows. To identify one interval \( t \) in different E-sets, let \( t^{(A)} \) refer \( t \) in E-set \( A \) (i.e. \( t^{(A)} = t^{(A)} \)). An interval \( t^{(B)} \) is said \( B \)-removable if \( t^{(B)} \) exists in \( B \) and is \( B\)-
removable.

The minimum OGI-set of $Z$ is derived by finding those intervals not removable in $X$ or $Y$, but becomes removable in $Z (= X \cup Y)$. The following Lemma demonstrates that the nonvolatile intervals remain nonremovable until some volatile interval becomes removable.

**Lemma 4.1** Let $Z = X \cup Y$ be the E-sets as described above. All the nonvolatile intervals in $F(X)$ become $Z$-removable if and only if some volatile interval in $F(X)$ becomes $Z$-removable.

**Proof.** ($\Rightarrow$) Consider a nonvolatile interval $t^{(X)}$ and a volatile interval $v^{(X)}$ in $F(X)$. Assume that $v^{(X)}$ is in process $P_k$. Figure 8(a) illustrates that $v$ is the last interval in $P_k$ that can overlap with $t$. (Otherwise, an interval $v'$ exists in $P_k$ such that $v'.lo \rightarrow t'.hi$, contradicts with $v$ is volatile in $X$.) This implies that if $v$ becomes $Z$-removable then $t$ also becomes $Z$-removable.

($\Leftarrow$) By contradiction, assume that all volatiles in $F(X)$ are $Z$-nonremovable whereas the nonvolatile $t$ is $Z$-removable. $F(X) = F(\tilde{Z})$ is proved as follows. For each ordered pair of $t_1, t_2 \in F(X)$ (note that $t_1^{(X)} \rightarrow t_2^{(X)}$), the property $t_1^{(Z)} \rightarrow t_2^{(Z)}$ holds because:

- If $t_2^{(X)}$ is nonvolatile (Figure 9(a)) then $t_1^{(X)} \rightarrow t_2^{(X)}$ implies $t_1^{(Z)} \rightarrow t_2^{(Z)}$.
- If $t_2^{(X)}$ is volatile and $t_2^{(Z)}$ remains volatile (Figure 9(b), clearly, $t_1^{(Z)} \rightarrow t_2^{(Z)}$ (see Definition 3.1).
- If $t_2^{(X)}$ is volatile but $t_2^{(Z)}$ becomes nonvolatile (Figure 9(c)), the property $t_1^{(Z)} \rightarrow t_2^{(Z)}$ is satisfied because: $t_1^{(X)} \in F(X)$, $t_2^{(Z)} \in F(\tilde{Z})$ (this is because $t_2^{(X)}$ is volatile in $F(X)$ and thus is $Z$-nonremovable by as-

Therefore, $F(X) = F(\tilde{Z})$, contradicts with the fact that $t \in F(\tilde{Z})$ is $Z$-removable. ■

Next, Lemma 4.2 and 4.3 demonstrate the condition for volatile intervals.

**Lemma 4.2** Let $Z = X \cup Y$ be the E-sets as described above. If neither $F(\tilde{X}) \leq F(Y)$ nor $F(\tilde{Y}) \leq F(X)$, then, $F(\tilde{Z}) = f(\tilde{Z})$.

**Proof.** If $F(\tilde{X}) \not\leq F(Y)$ and $F(\tilde{Y}) \not\leq F(X)$, there exist intervals $t_1, t_2 \in F(\tilde{X})$ and $u_1, u_2 \in F(\tilde{Y})$ such that $\{t_1\} \prec \{u_2\}$ and $\{u_1\} \prec \{t_2\}$. Therefore, $t_1$ and $u_1$ are nonvolatile in $Z$ and is $Z$-removable. From Lemma 4.1, all nonvolatile intervals in $F(X)$ and $F(Y)$ are $Z$-removable. Thus, $F(\tilde{Z}) = f(\tilde{Z})$. ■
Let $Z = X \cup Y$ be the E-sets as described above. Furthermore, assume that $F(\hat{Y}) \leq F(\hat{X})$. A volatile interval $t(X) \in F(\hat{X})$ becomes $Z$-removable if and only if $t(Y)$ is non-volatile and the following properties are satisfied:

1. $t(Y)$ is $Y$-removable, or
2. $t(Y)$ is $Y$-nonremovable and some volatile interval $u \in F(\hat{Y})$ is $X$-removable.

Proof. In this proof, only the second case, i.e. $t$ is $Y$-nonremovable, is considered. (In the first case, clearly, $t$ is $Y$-removable implies that $t$ is $Z$-removable.)

$(\Rightarrow)$ If $u$ is $X$-removable, from Lemma 4.1, all the nonvolatile intervals in $F(\hat{Y})$, including $t(Y)$, are $Z$-removable.

$(\Rightarrow)$ First, $t(Y)$ must be nonvolatile from Property P2. By contradiction, assume that $t$ is $Z$-removable but $t(Y)$ is $Y$-nonremovable and all the volatile intervals $u \in F(\hat{Y})$ are $X$-nonremovable. This assumption implies that $F(\hat{X}) = F(\hat{Y})$ (the detail is ignored since the proof is very similar to Lemma 4.1). Since $Z = X \cup Y$, $F(\hat{X}) = F(\hat{Y})$ is an OGI-set in $Z$. This implies that $F(\hat{Z}) = F(\hat{X}) = F(\hat{Y})$, contradict with the fact that $t \in F(\hat{X}) = F(\hat{Z})$ is $Z$-removable.

## 4.3 The New Detection Algorithm

In our new detection algorithm, each process keeps only two $p$-tuple vectors $F$ and $f$ to represent the minimum OGI-set and the volatile interval set of this process, respectively. This algorithm consists of the following procedures that are executed at each process $P_i$:

- **Procedure** InternalEvent (to be called when $P_i$ executes an internal event, say $e_{i,x}$):
  1. After execution of event $e_{i,x}$, the truth value of $LP_i$ may change, as follows:
     a. The truth value of $LP_i$ is unchanged: Both $F$ and $f$ are remain unchanged.
     b. The truth value of $LP_i$ is in a false-to-true transition: Modify $f[i]$ to the new volatile interval whose beginning event is $e_{i,x}$.
     c. The truth value of $LP_i$ is in a true-to-false transition: In this case, the current volatile interval of process $P_i$ is ended at event $e_{i,x}$. Modify $f[i]$ to the new volatile interval in which both beginning and end events are pseudo.

- **Procedure** SendEvent (to be called when $P_i$ executes a send event, say $e_{i,x}$):
  1. Since the truth value of $LP_i$ is unchanged, both $F$ and $f$ are also unchanged.
  2. Piggyback $F$ and $f$ in the message and then send it.

- **Procedure** ReceiveEvent (to be called when $P_i$ executes a receive event, say $e_{i,x}$):
  1. Assume that the sent event of this message is $e_{j,y}$. Receive the message and extract data $F^i$ (represent $F(\tilde{E}_{j,y})$) and $f^i$ (represent $f(\tilde{E}_{j,y})$).
  2. Let $f[k] = \max(f[k], f'[k])$, $k = 1, 2, \ldots, p$.
  3. (Based on Lemma 4.2) If $F(\tilde{E}_{j,y}) \not\subseteq F(\tilde{E}_{j,y})$ or $F(\tilde{E}_{j,y}) \not\subseteq F(\tilde{E}_{j,y})$ then $f = f^i$. Go to Step 7.
  4. If $F(\tilde{E}_{j,y}) \leq F(\tilde{E}_{j,y})$, determine whether an interval $t$ is $\tilde{E}_{i,x}$-removable as follows:
     a. (Based on Property P3) If $t \prec F(\tilde{E}_{i,x})$ then $t$ is removable.
     b. (Based on Lemma 4.3) If $t \in F(\tilde{E}_{i,x})$ and $t$ is volatile:
        i. If the following properties are satisfied, mark $t$ as removable:
           A. $t$ is $\tilde{E}_{j,y}$-removable, or
           B. $t$ is $\tilde{E}_{j,y}$-nonremovable and some volatile interval $u \in F(\tilde{E}_{j,y})$ is $\tilde{E}_{i,x}$-removable. For example, consider an interval $t_2$ illustrated in Figure 10.
              The interval $t_2$ is $\tilde{E}_{j,y}$-nonremovable and volatile interval $t_1 \in F(\tilde{E}_{j,y})$ is $\tilde{E}_{i,x}$-removable (Figure 10(a) and (b)). Hence, $t_2$ is $\tilde{E}_{i,x}$-removable (Figure 10(c)).
        c. (Based on Lemma 4.1) If $t \in F(\tilde{E}_{i,x})$ and $t$ is nonvolatile:
i. If any volatile interval is marked as removable in Step 4(b), mark \( t \) as removable.

5. If \( F(\hat{E}_{i,x-1}) \leq F(\hat{E}_{j,y}) \), repeat Step 4 except that the roles of \( F(\hat{E}_{i,x-1}) \) and \( F(\hat{E}_{j,y}) \) are swapped.

6. Let \( F(\hat{E}_{i,x}) \) be the set of intervals that not have been marked as removable in previous steps.

7. If \( F(\hat{E}_{i,x}) \) contains no volatile then \( \text{DEFINITELY}(\Phi) \) is true. Otherwise, \( \text{DEFINITELY}(\Phi) \) is false.

In this algorithm, a process cannot evaluate the global predicate if there are no messages sent from other processes to carry the debug information. To solve this problem, if \( \text{DEFINITELY}(\Phi) \) is still false at the end of the program execution, \( p \) extra messages are sent among all \( p \) processes in a circular way to pass the debug information. However, as compared with the cost of the entire distributed computation, these \( p \) messages incur a very low overhead.

The correctness of this algorithm can be verified easily based on the theorems presented in Subsection 4.2. Before analyzing the complexity of the algorithm, the implementation has to be explained. First, vectors \( F \) and \( f \) are implemented by using vectors of integers: when process \( P_i \) executes the event \( e_{i,x} \), the value \( F[j] \) (resp. \( f[j] \)) equals the sequence number of the beginning event of interval \( F(\hat{E}_{i,x})[j] \) (resp. \( f(\hat{E}_{i,x})[j] \)). The operations of the algorithm is implemented as follows:

- \( F(\hat{E}_{j,y}) \leq F(\hat{E}_{i,x-1}) \) iff \( F'[k] \leq F[k] \), for all \( k \) (assume that \( F' \) refers to \( F(\hat{E}_{j,y}) \) and \( F \) refers to \( F(\hat{E}_{i,x-1}) \)). This operation takes \( O(p) \) time.
- Interval \( F[k] \) is volatile iff \( F[k] = f[k] \). This operation takes \( O(1) \) time.
- Interval \( t \) with \( t.lo = e_{k,z} \) is \( E_{i,x} \)-removable if \( t \) is nonvolatile and \( F[k] \neq z \), (assume that \( F' \) refers to \( F(\hat{E}_{i,x}) \)). This operation takes \( O(1) \) time.

Based on the above implementation, each invoke of the procedures (InternalEvent, SendEvent, and ReceiveEvent) requires \( O(p) \) time. Assume that
there are \( n \) events for process \( P \), the total time complexity for the process is \( O(n^2) \).

5 Discussion

This paper investigates the problem of detection of definitely true conjunctive predicates (DEFINITELY(\( \Phi \))). To solve the problem of unbounded queue growth of previous detection algorithms, in this paper, the notion of removable states is introduced. By discarding the removable states, the space requirement for each process can be minimized to \( O(p) \), where \( p \) is the number of processes. While bounding the memory space, this analysis shows that the time complexity of the proposed algorithm is only \( O(pm) \), faster than previous algorithms are, by a factor of \( p \).

Another related detection problem regarding to the distributed debugging is to detect what possibly true conjunctive predicates (POSSIBLY(\( \Phi \))) [7, 11, 12]. To enhance the performance of POSSIBLY(\( \Phi \)) detection algorithms, Chiou and Korfhage [3] presented two algorithms to remove some useless states for the detection. However, their algorithms run in a centralized environment and identify only partial useless states. For the distributed detection of POSSIBLY(\( \Phi \)), finding an efficient approach to identify the removable states is still a research topic.

References


